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CONVAIR ASTRONAUTICS

CONVAIR DIVISION OF GENERAL DYNAMICS CORPORATION

ZERO-G REPORT

LIQUID BEHAVIOR SCALING

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REVISIONS

NO.	DATE	BY	CHANGE	PAGES AFFECTED
A	7/16/62	CKP CKP	Equation (a)	3

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1.0 INTRODUCTION:

The complexity of the behavior of moving liquids --- even in zero-g --- tends to make analytical solutions very tedious or even to rule them out entirely. The size of the Centaur tank makes zero-g full scale tests impossible before the vehicle is ready for use and very expensive afterwards. The well known approach to a problem like this is the use of scaled down models. The model, the conditions under which the tests are executed, and the transformation of the results from model size to full size are governed by appropriate scaling laws or similitude requirements. This is obviously of great interest and can give much design information in a relatively inexpensive way.

In the spring of 1961 we were planning an airplane zero-g experimental investigation of liquid behavior. This seemed like an excellent opportunity for saving some money and trouble, and we started to do theoretical work on scaling principles. The results were so promising that further airplane tests on liquid behavior were obviated.

The first developments of the scaling principles are outlined in Appendix A, which is a full copy of Memo 562-1-550, dated 4-25-61. We have since verified and corroborated those laws and used them for various purposes in the Centaur program.

2.0 CORROBORATION:

We found (Appendix A) the following expressions for the oscillatory motion of a fluid

(1) $T = C_1\sqrt{\rho/\sigma \; D^3}$

and

(2) $\tau = C_2 \; \rho/\mu \; D^2$

where:

T = period of oscillation

τ = time required for the amplitude of the oscillation to dampen to a certain fraction of its original value

D = characteristic dimension (e.g. diameter)

ρ = density of the liquid

μ = viscosity of the liquid

σ = surface tension of the liquid

C_1, C_2 = dimensionless constants depending on the nature of the oscillating system

These expressions were corroborated in several ways (in addition to the analytical deduction by Sir Horace Lamb).

2.0 CORROBORATION: (Cont)

2.1 **Model laws:**

From the well known similitude requirements for hydraulic models we have

(a) Webers number

$$We = \frac{\rho L V^2}{\sigma}$$

(b) Reynolds number

$$Re = \frac{LV}{\mu/\rho}$$

Where L is a characteristic dimension (length), and V is the fluid velocity.

When it is remembered that velocity is length divided by time it is easily seen that our equation (1) essentially says that the oscillation time is scaled in accordance with the similitude requirements imposed by Webers number, and that our equation (2) essentially says that the damping is scaled in accordance with the similitude requirements imposed by Reynolds number.

2.2 Experiments with Liquid/Liquid Models:

Equation (1) was carefully verified with experiments with globules of one liquid oscillating in a large amount of another (immiscible) liquid balanced to the same density (this cancels the effect of gravity, making for this purpose a zero-g environment).

2.0 CORROBORATION: (Cont)

2.2 Experiments with Liquid/Liquid Models: (Cont)

The test results matched the theory to within one percent, which was the accuracy limit of the test method.

Figure No. 1 shows the results using water globules in a mixture of Freon TF and Stoddard solvent, and kerosene globules in a mixture of water and methyl alcohol.

The curved line, marked 35th, shows the influence of confining walls (in this case a 1/35 scale Centaur liquid hydrogen tank) on the oscillation times. The percentages indicate % ullage in the tank.

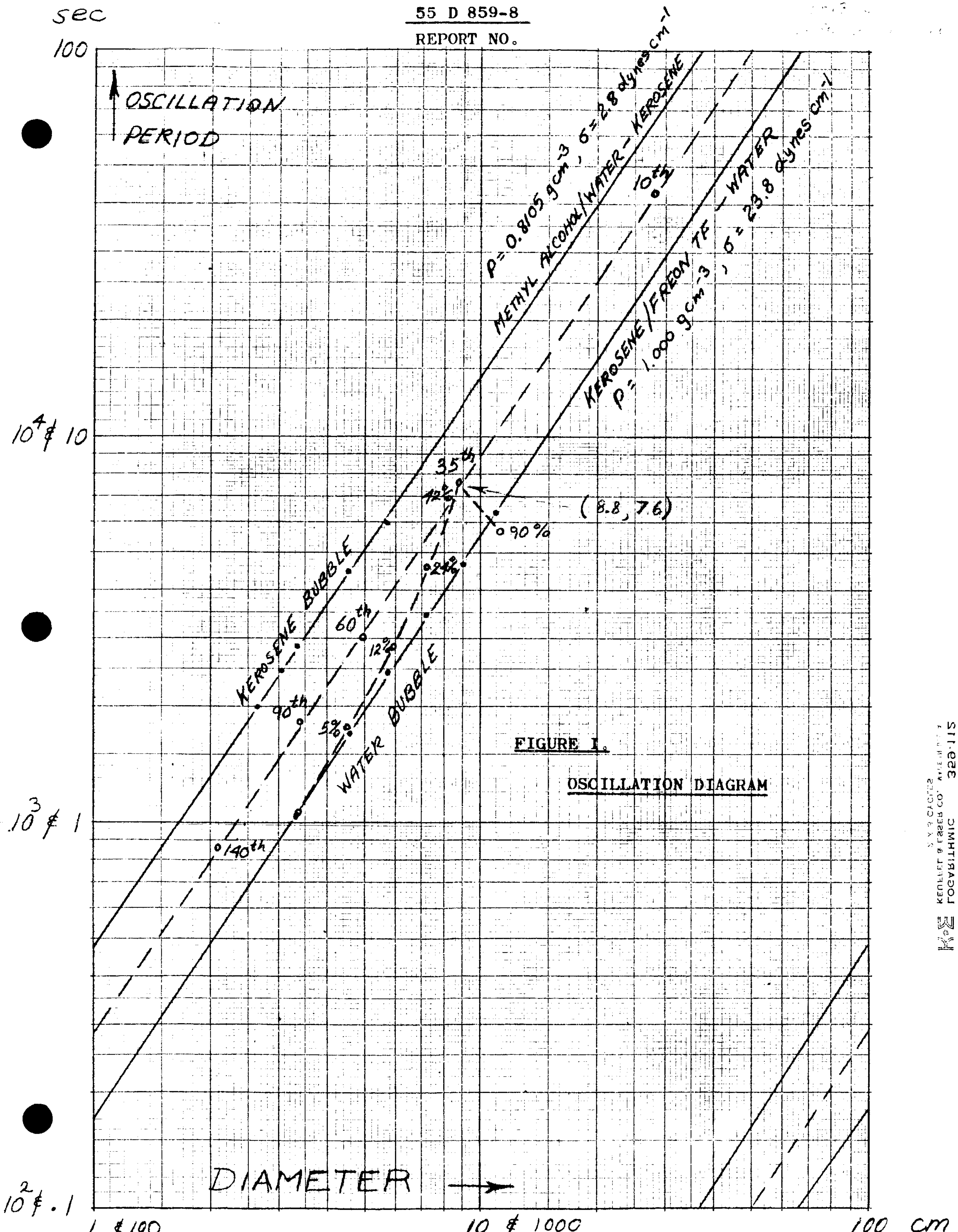


FIGURE I.

OSCILLATION DIAGRAM

K&E KEUFFEL & ESSER CO. LOGARITHMIC 359-115

<u>APPENDIX I:</u>

Revised: 4/25/61
Retyped: 5/7/62

TO: E. W. Schwartz/J. Elizalde, Dept. 595-1

FROM: G. B. Wood, Dept. 562-1

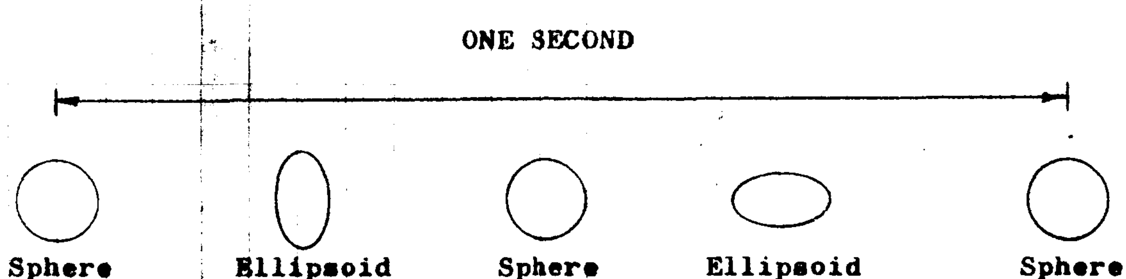
SUBJECT: Liquid Oscillations and Damping

<u>INTRODUCTION:</u>

During the Centaur Zero-G flights in a KC-135 airplane early in 1961 it was observed that LN_2 often escaped from a cold trap on the LH_2 storage dewar. Most of the time the drops that formed (usually a jet would hit the ceiling and split up) were small, and they oscillated very rapidly and stabilized quickly to perfect spheres as they floated around in the cabin.

Once in awhile a bigger drop would form by chance without hitting the ceiling and could be observed in free floating condition somewhat better. It would have a rapidly fluctuating wrinkly appearance at first but would soon smooth out some and oscillate in a more regular way.

It was estimated on one occasion that a drop of approximately one inch diameter made a complete oscillation in the basic mode in roughly one second.



It was thought that those observations could be used for establishment of approximate scaling laws for liquid behavior.

APPENDIX I: (Cont)

Analysis: (Cont)

A bit of library research uncovered corroboration for the dimensional analysis in Sir Horace Lamb's "Hydrodynamics" --- first published in 1879. Sections 275 and 355 of that work present analyses of and equations for the motion of oscillating fluid globules. The relations are shown below.

(3)
$$T = \pi \sqrt{\frac{(n+1)f_g + n f_s}{2n(n+1)(n-1)(n+2)} D^3}$$

Where, in addition to the previously defined variables, n = the harmonic order of the oscillation. ($n = 2$ for the simplest vibratory mode)

$f_g = \rho_g / \sigma$

$f_s = \rho_s / \sigma$

ρ_g = density of fluid in the globule

ρ_s = density of fluid surrounding the globule

σ = surface tension of the globule/surround interface.

Also, for a globule (surrounded by a fluid of negligible ρ and μ).

(4)
$$T = \frac{1}{4(n-1)(2n+1)} \phi_g D^2$$

And, for a bubble (filled with a fluid of negligible ρ and μ)

(5)
$$T = \frac{1}{4(n+2)(2n+1)} \phi_s D^2$$

Where T is now elevated to the status of "Modulus of Decay", which is the time in which the amplitude of oscillation sinks to $1/e$ of its original value.

APPENDIX I: (Cont)

Analysis: (Cont)

For $m = 2$ the above equations readily reduce to:

(6) $T = \frac{\pi}{4}\sqrt{f D^3}$

for a globule

$T = \frac{1}{20}\phi D^2$

and,

(7) $T = \frac{\pi}{\sqrt{24}}\sqrt{f D^3}$

for a bubble

$T = \frac{1}{80}\phi D^2$

CONCLUSION:

The simple vibrating systems described by the above equations bear little direct relation to the complex shapes and surfaces which are usually seen in zero gravity, or to those which will probably exist in the Centaur fuel tank. The simple systems do, however, have the feature in common with the complex ones that their response to a disturbance is controlled by ρ , μ , and σ . The analytical expressions corroborate, for the simple system, the dimensional analysis which should also apply to the complex systems; the one data point lies remarkably close to the proper LN_2 line, and it appears therefore that the dimensional scaling laws presented here are very nearly the truth.

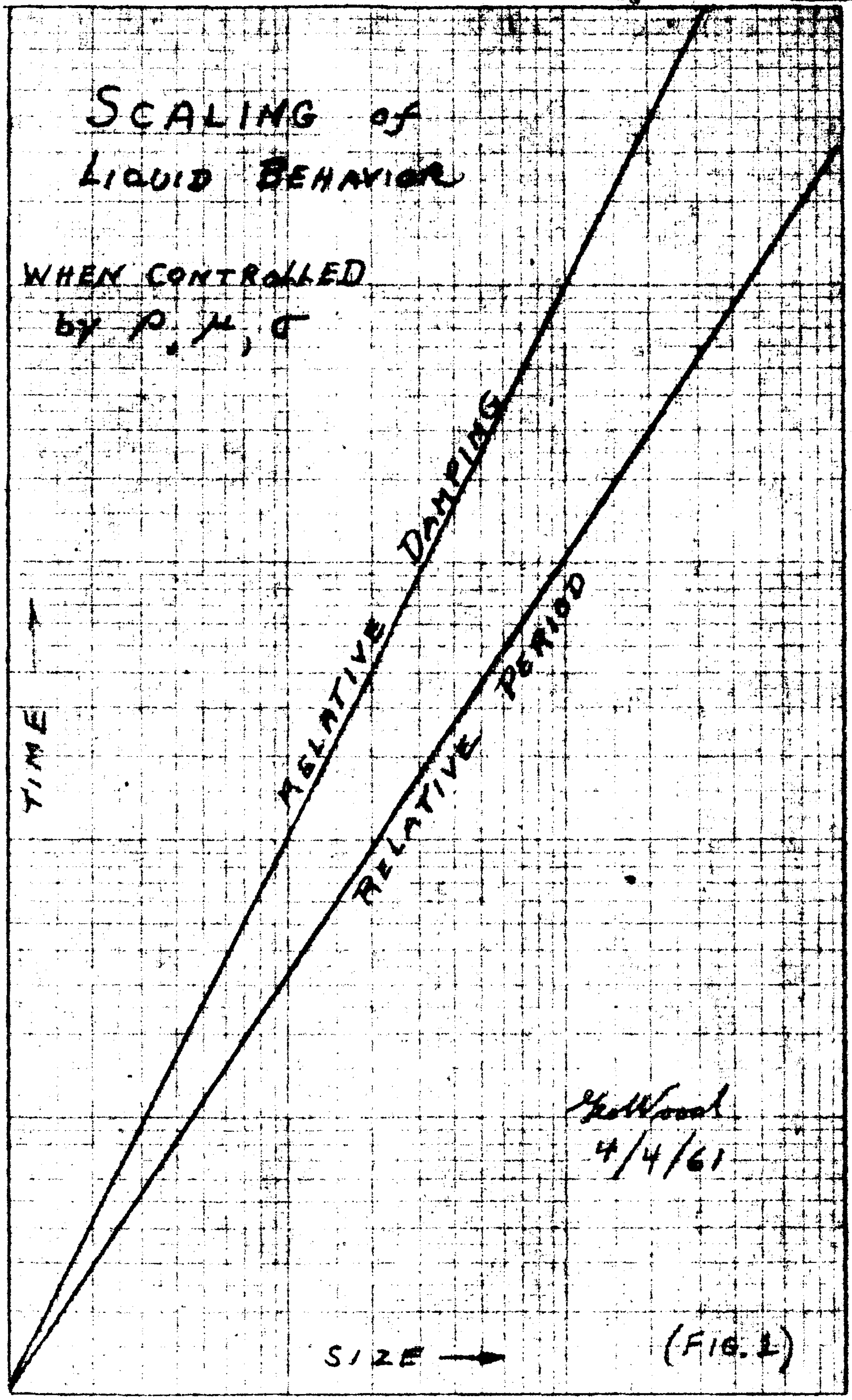
Certain liquid data is shown in Table "A", and equations (6) and (7) are graphed in Figure 2.

J. L. Hansen

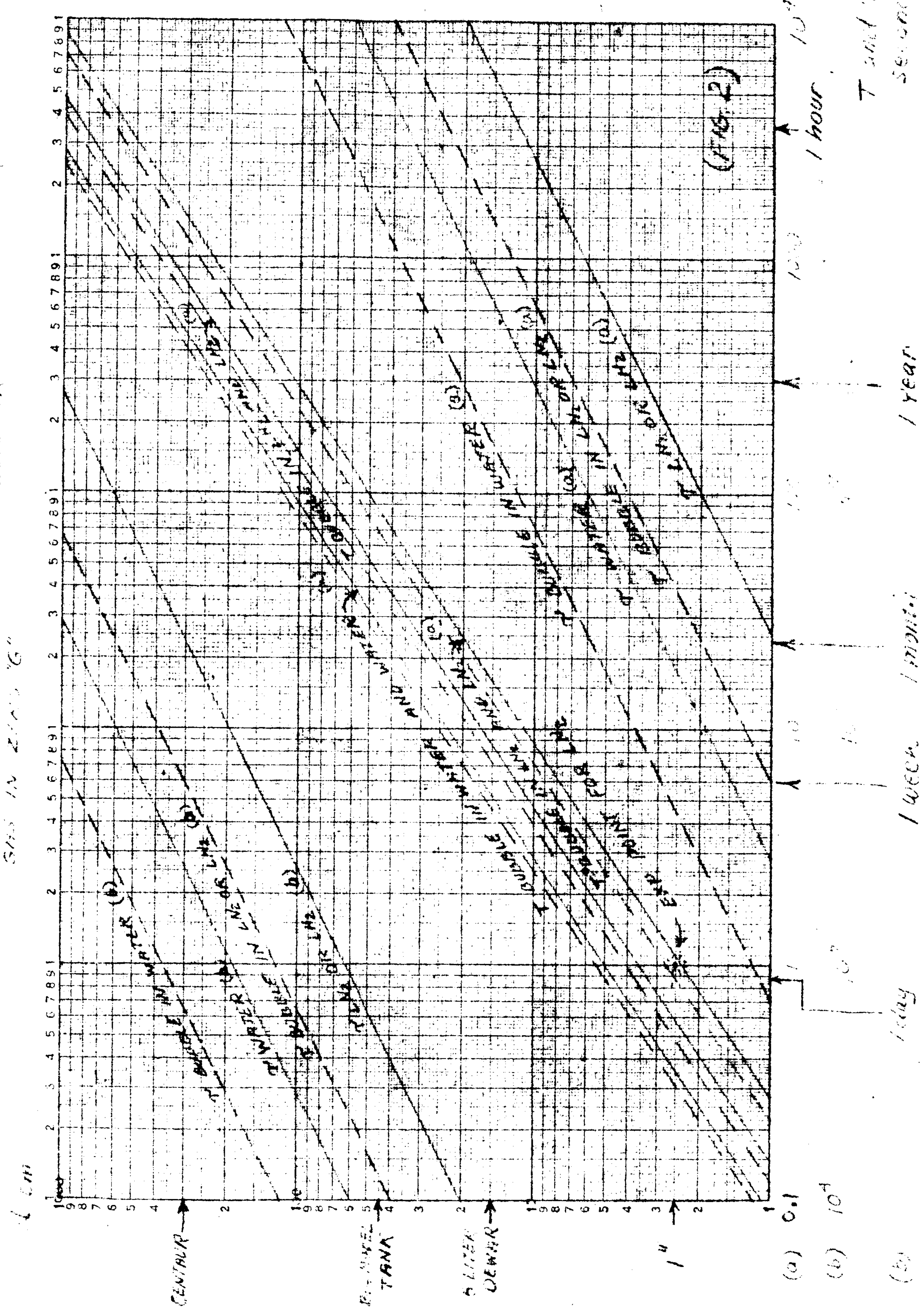
G. B. Wood,
Design Specialist

TABLE "A"

Fluid (Liquid)	ρ (g/cm^3)	σ (dy/cm)	μ $\left(\frac{dy\ sec.}{cm^2}\right)$	f $\left(\frac{sec^2}{cm^3}\right)$	ϕ $\left(\frac{sec}{cm^2}\right)$
H_2O	1	73.0	.0178	.0137	56.2
LN_2	.81	6.6	.0016	.123	506
LH_2	.070	2.0	.000135	.035	518
Freon TF $(CCl_2F-CClF_2)$	1.57	19.0	.00694	.083	226


SCALING of
LIQUID BEHAVIOR
WHEN CONTROLLED
by ρ, μ, σ
RELATIVE DAMPING
RELATIVE PERIOD
TIME
SIZE
4/4/61

(FIG. 1)


LOGARITHMIC 359-125G
(FIG. 2)
1 hour
1 year
1 week
1 day
CENTAUR
TANK
DEWAR
T in seconds